

Fig. 3 Ratios of critical external compressive stress,  $S_c$ , for elastic yielding of circular sections of the indicated pairs of materials. Comparative data provide an approximation to behavior of copper and lead coils relative to MgO as a pressurizing material. Ideal combination of materials in that yielding a ratio of unity



Fig. 4 Ratios of critical external compressive stress,  $S_c$ , for elastic yielding of identical thin rings of halite and lead, and halite and copper, respectively. Relative to MgO as shown in Fig. 3, the much weaker NaCl yields ratios which straddle unity. Pb is preferred coil material to use with halite as a pressurizing material since its ratio most closely approaches unity, and especially since it does so from high side which would assure compliance of wire to core

shown later that measures (c), (d), and (e) are to be preferred.

Fig.2 provides a rather typical illustration of response to initial deformation. This experiment was performed in a multianvil apparatus of cubic design (4). The experimental assembly consisted of a threaded tungsten core with a handwound copper coil which was enclosed within a thin protective pyrophyllite sleeve. The assembly was enveloped in silver chloride and placed within a standard pyrophyllite block (2). The pressure range shown is about 15 kilobars. The interpretation of coil behavior is as follows: (a) Initial



Fig. 5 Ratios of critical external compressive stress for elastic yielding. Combinations, AgCl versus Pb and AgCl versus Cu, represent the case for a pressurizing material very prone to plastic flow. Of two combinations given, Pb is preferred coil wire for use in silver chloride since its  $S_c$  ratio provides closest approach to unity. Note projected crossover at about 50 kb

compression of the coil; (b) yield, collapse, and compliance to groove geometry; (c) deformation (also a transitory pressure drop) owing to gasketforming displacements, and (d) stabilization and full coupling to the core.

Semiquantitative support for the interpretation of coil collapse in various environments can be had by calculation and comparison of critical elastic yield and buckling pressures of both coil and pressurizing materials in the form of identical thin circular sections. The equations which have been used are: For the critical external compressive stress for elastic yielding,  $S_c$ 

$$S_{c} = \frac{k E t^{2}}{(1 - e^{-2}) R_{0}^{2}}$$

where k = the radial ratio factor, t = the thickness of the ring wall,  $R_0$  = the outer radius, E = Young's modulus,  $\sigma$  = the Poisson ratio for the material. The equations used to compare buckling pressures are

$$P_c = \frac{3 E I}{R^3}$$
, for elliptical deformation, and  
 $P_c = \frac{15 E I}{R^3}$ , for a four lobed distortion.

I = the moment of the ring. Ratios of critical elastic yield pressures for combinations of MgO, NaCl, and AgCl with Cu and Pb, respectively, to high pressures are given in Figs.3, 4, and 5. Ratios greater than unity indicate preferential

yielding of the coil, whereas less than unity signify early yield of the environmental material. Ideally, unit ratio is sought.

Sources of error discussed thus far are straightforward and can be essentially eliminated (< 2 percent) without serious difficulty. An additional but less obvious source of error is the existence of internal pressure differences within layered multicomponent assemblies under external pressure. The causes of such pressure differences are relative differences in the elastic properties and the geometrical relationships of the respective solids. This type of error is of general significance.

Clarification of the existence of internal pressure differences automatically extends the problem to the basic mechanics of establishing and measuring pressure intensities within solids!

The magnitude of error which can result from an unappreciated presence of internal pressure gain or attenuation can be large. Depending on the materials present in a given assemblage, and their relative dimensions, differences as large as 50 percent have been observed. With the exceptions of Bobrowsky (5), Giardini (6), and Corll and Warren (7), the existence of inherent pressure variation within nonhomogeneous solid systems has been essentially ignored by high-pressure researchers.

Experiences with the inductive-coil technique have provided additional strong qualitative support for their occurrence. Specific quantitative experiments remain to be carried out, however, it is now reasonable to suspect that "selfgenerating" internal pressure differences constitute one of the major sources of error, and therefore irreproducibility, in most high-pressure work carried out in solid environments.

Analytical evaluation of the problem of internal pressure differences can be approached by applying classical elastic theory to the case of a specimen in right circular cylindrical form enclosed by a cylindrical shell of a different solid which is exposed to an external hydrostatic pressure. The objective is calculation of the radial pressure profile of the assembly. Following the method of Bobrowsky (5), the analysis is based on the Lamé equation which describes the radial displacement of an elastically isotropic cylindrical shell subjected to hydrostatic pressure

$$\frac{\mathbf{U}_{R}}{\mathbf{E}} = \frac{(1 - \mathbf{e}) (R_{1}^{2} P_{1} - R_{e}^{2} P_{e}) R}{E (R_{e}^{2} - R_{1}^{2})} + \frac{(1 + \mathbf{e}) R_{1}^{2} R_{e}^{2} (P_{1} - P_{e})}{E (R_{e}^{2} - R_{1}^{2}) R}$$

U<sub>R</sub> is the radial displacement at any radius, R;

 $P_e$  is the external hydrostatic pressure;  $P_i$  is the internal hydrostatic pressure;  $R_e$  is the initial external radius of the cylindrical shell; and  $R_i$  is the initial internal radius; E is the Young modulus of elasticity; and *a* is the Poisson ratio of the shell material.

Taking the Lamé equation and setting  $R_i$  equal to zero for the case of a solid cylindrical core appropriate to the inductive-coil core, we obtain the following equation to describe its radial displacement:

$$U_{R_e} = -P_e R_e \frac{(1 - e^{-})}{E}$$

We set  $R_i$  of the shell equal to  $R_e$  of the enclosed core. The displacement of this mutual interface must of necessity be equal in response to an externally applied pressure,  $P_e$ , as long as the interface remains insulated from the pressure media of  $P_e$ . Consequently, by setting  $U_R$  of the cylindrical shell equal to  $U_{R_i}$ , and equating  $U_{R_i}$ of the shell to  $U_{R_e}$  of the core, we have

$$\frac{P_{i} R_{i} (1 - e^{-i})}{E^{i}} = \frac{(1 - e^{-i})}{E} \frac{(R_{i}^{2} P_{i} - R_{e}^{2} P_{e}) R_{i}}{(R_{e}^{2} - R_{i}^{2})} + \frac{(1 + e^{-i})}{E} \frac{R_{i}^{2} R_{e}^{2} (P_{i} - P_{e})}{R_{i} (R_{e}^{2} - R_{i}^{2})}$$

The primed constants are those of the core material when it is different from that of the enclosing shell. Rearranging terms, the following equation is obtained:

$$\frac{E (1 - \sigma^{-1}) (R_e^2 - R_1^2)}{P_1} = \frac{-E' ((1 - \sigma^{-}) R_1^2 + (1 + \sigma^{-}) R_e^2)}{2 E' R_e^2}$$

The expression  $P_e/P_i$  gives in dimensionless form the numerical relationship between the externally applied hydrostatic pressure,  $P_e$ , and the resultant pressure,  $P_i$ , which the core experiences on its periphery. If both segments of the system are of the same material, the ratio is equal to unity and thus establishes the validity of the equation. If the materials are different and possess different elastic constants, then  $P_e/P_i$  will be different from unity unless compensation can be achieved by manipulation of the dimensional relationships of the components. The divergence of the ratio from unity provides the magnitude and direction of the pressure difference.

The following assumptions are explicit for quantitative validity of the equation:

1 The cylindrical assembly is symmetrical